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LARGE GAS TO GAS THERMOELECTRIC HEAT PUMPS

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Summary
Cross flow thermoelectric heat pumps are examined mathematically. The mathematical model is used to determine the extreme conditions where the coefficient of performance is near to unity. A system that operates in both heating and cooling mode is presented. The influence of various parameters such as electrical current density, lengths of the primary and auxilary gas circuits, flow rates, thermoelectric material thickness and properties on performances is studied. Frosting conditions are evaluated.

1 - Mathematical model

Thermoelectric units, especially of the cross flow type, are easily assembled in parallel to form systems. The model corresponds to a unit shown in Fig. 1.

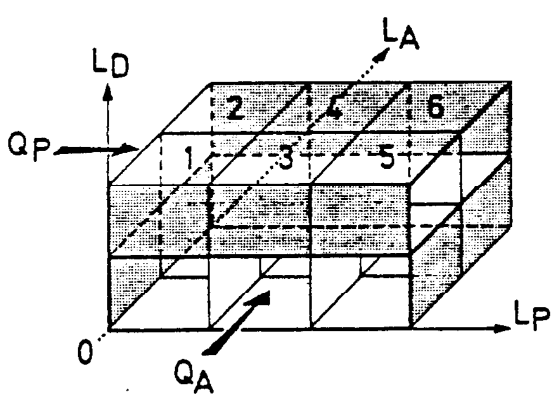

Fig. 1 - Schematic of a unit composed of 6 subunits.

The unit is divided into subunits. The subunits are calculated one by one with the following equations :

$$P_H = +aI\,(T+273.15) + \frac{1}{2}\, r\, I^2 - C_E(T_H - T_C) - C_L(T_{HB} - T_{CB}) \quad (1)$$

$$P_C = - aI(T_C+273.15) + \frac{1}{2}\, r\, I^2 + C_E(T_H - T_C) + C_L(T_{HB} - T_{CB}) \quad (2)$$

The indices are H for hot side and C for cold side. In the following equations the indices H and C will not be written, nevertheless each of the equations is duplicated one for the hot side and one for the cold side.

Fig. 2 is a schematic of a subunit showing the different temperatures and thermal conductance terms.

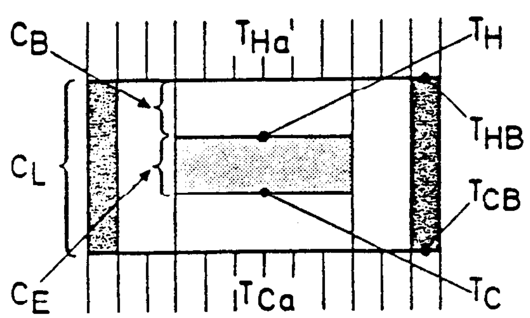

Fig. 2 - Schematic detail of subunit.

T are the temperatures at the interfaces of the thermoelectric material.

T_B are the temperatures at the base of the heat exchangers.

Ta are gas temperatures.

$$T = Ta + \frac{P}{C} = T_B + \frac{P}{C_B} \quad (3)$$

$$T_B = Ta + \frac{P}{Ca} \quad (4) \qquad Ta = T_{a1} + \frac{P}{2.6\,\varpi} \quad (5)$$

The terms T and T_B of equations (1) and (2) can be replaced by Ta.

G = mass gas flow rate

ϖ = heat capacity of gas

C_L = Thermal conductance corresponding to heat losses between the hot and cold heat exchangers excluding the thermoelectric material.

C = Thermal conductance of heat exchanger, it has two components :

- C_B - Conduction term in the solid from interface of thermoelement component.
- C_a - Convection term between surface of heat exchanger and gas.

Equations (1) to (5) are used to calculate each subunit knowing :

- Inlet conditions of both circuits.
- Electrical current going through the thermoelectric material.

The calculation is started at subunit 1 see Fig. 1 where the inlet conditions are known, the heating, cooling and electric powers involved are calculated and also the exit conditions which enable the calculation to be repeated for subunit 2 and so on.

2 - Parameter analysis

A calculation like the one described in the previous paragraph gives the heating power for a given set of parameters. As soon as one parameter is changed, it is very difficult to make comparisons, for this reason the following approcach is an attemps at presenting results with a maximum of non dimensional parameters so that the performances of different units can be plotted on the same diagram.

2.1. General parameters

The parameters can be grouped into 5 categories :

1) Electrical and thermoelectric material characteristics
   - a, ρ, λ must be known versus temperature
   - Quantity of material
     . Surface area S_E (perpendicular to electrical current)
     . Thickness e (parallel to electrical current)
   - Electrical resistance r_L in a unit excluding the thermoelectric material.
2) Overall dimensions
   - Length of primary circuit L_P
   - Length of auxilary circuit L_A
   - Length of 3rd dimension L_D
3) Heat exchanger characteristics
   - Surface areas in contact with the gas
   - Thermal conductance C_B between interfaces with thermoelectric component and fin exchanger surface

- Heat transfer law H between fin surface S and the gas. It can be expressed as a function of gas velocity.

$$H = M \cdot V^N \qquad (6)$$

4) Operating parameters

- Electrical current density J $A.cm^{-2}$
- Mass flow rate of each gas G_P and G_A

5) Inlet conditions

- Inlet temperatures T_{Pa1} ; T_{Aa1}
- Inlet enthalpies : for air it is sufficient to indicate the humidity for example relative humidity.

It becomes very complicated when all the above parameters are given in detail so certain parameters have been grouped together.

2.2. Non dimensional parameters

The following non dimensional parameters are useful

$$\frac{L_P}{L_A} = \frac{\text{Length of primary gas circuit}}{\text{Length of auxilary gas circuit}}$$

When the two velocities of each circuit remain unchanged, it is not necessary to study separately the variations of $\frac{L_A}{L_P}$ and $\frac{Q_P}{Q_A}$.

$$\frac{G_P}{G_A} = \frac{\text{Primary gas mass flow rate}}{\text{Auxilary gas mass flow rate}}$$

$$P^* = \frac{P_H}{Q_P \cdot \Delta P_P} = \frac{\text{heating power}}{\text{volume flow rate x pressure drop (primary circuit)}}$$

This is the ratio of available heating power to the power required to flow the primary gas through the circuit.

2.3. Power parameters

Five parameters with the dimensions of a power are defined.

- Aerodynamic energy losses

$\overline{Q}_P = Q_P . \Delta P_P$ primary circuit

$\overline{Q}_A = Q_A . \Delta P_A$ auxilary circuit

- Electrical Joule losses

$\overline{R} = r\ I^2$

This term has two components :

$\overline{R}_E$ thermoelectric material $\rho . v_E . J^2$

$\overline{R}_L$ connection losses $r_L . I^2$

- Peltier power

$\overline{P} = a\ I\ (T + 273.15)$ can be expressed as

$= a . S_E . J . (T + 273.15)$

where S_E = surface of thermoelectric material

- Thermal losses

This term has two components :

$\overline{C}_E$ = Losses thermoelectric material = $C_E\ (T_H - T_C)$

$\overline{C}_L$ = Losses outside thermoelectric material

$= C_L\ (T_{HB} - T_{EB})$

3 - Influence of parameters

3.1. Parameter $L_P \cdot L_A^{-1}$

The diagram of $P^* = P_H \cdot Q_P^{-1} \cdot \Delta P_P^{-1}$ versus Joule losses $\overline{R}_E$ through the thermoelectric material and the connections characterizes well a unit.

For example Fig. 3 gives $\overline{P}$ versus $\overline{R}$ for several values of $L_P . L_A^{-1}$ one notes that the variation of P^* when L_P goes from 2 to 6 is less than 3 %.

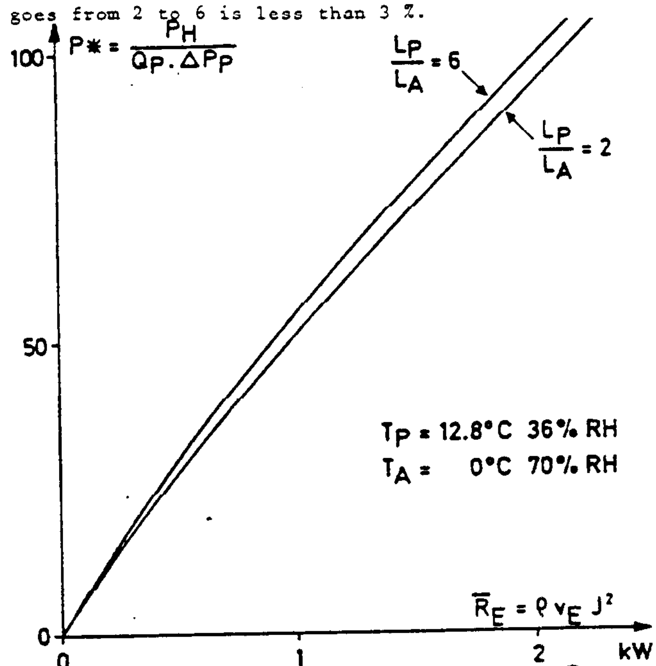

Fig. 3 Influence of $L_P . L_A^{-1}$ on heating power P^*.

Figure 4 gives P^* versus $\overline{R}$ with $L_P . L_A^{-1}$ = constant.

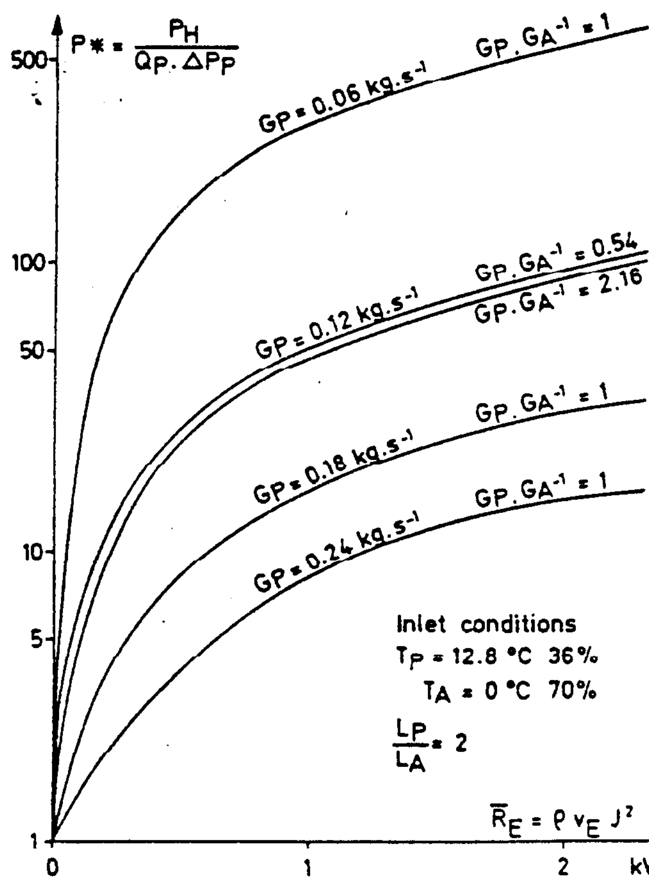

Fig. 4 - Influence of G_P and $G_P . G_A^{-1}$ on heating power P^*

The above graph shows that for a given configuration with LP/LA = constant, and given inlet temperature and humidity conditions, the non dimensional heating power P^* varies essentially with the primary mass gas flow rate GP : P^* varies in practice from 2 to 700. The ratio $\frac{GP}{GA}$ when varied between 0.5 and 2.2 has an influence on P^* of between 6 and 13 %, this is shown only for G_P = 0.12 kg s^{-1}. For the other values of G_P the curves correspond to $\frac{GP}{GA} = 1$.

3.3. Influence of the thickness of the thermoelectric material

The thickness e of the thermoelectric material is an important parameter because as e increases the Joule heating increases and the thermal conductivity across the material decreases.
Figure 5 shows in dashed lines the variations of P_H and in solid lines COP as a function of the material thickness e in mm. The curves are parametered in electrical current density J (A.cm2).

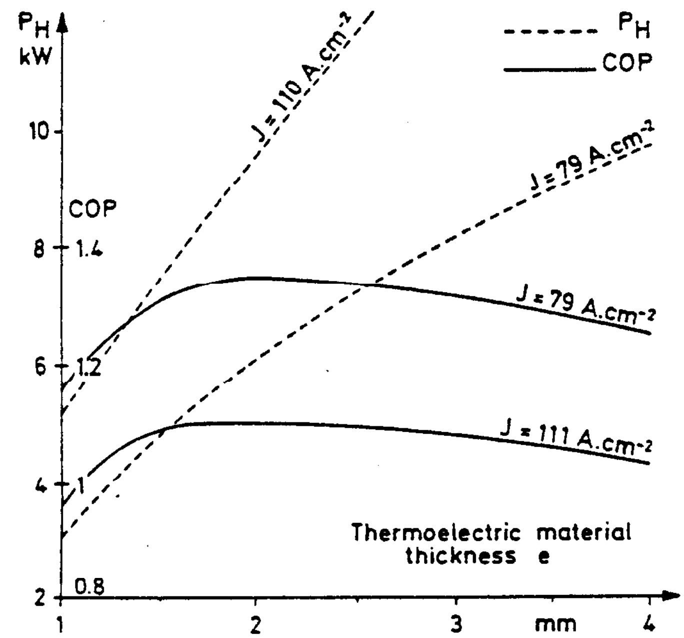

Fig. 5 - Influence of thermoelectric material thickness on heating power P_H and COP.

The optimum for this system is around 2 mm, but an increase in material thickness increases the heating power without a measurable reduction in COP. This can be of interest for systems that operate essentially in the heating mode.

3.4. Influence of material characteristics

The coefficient of merit Z of a thermoelectric material is most widely used to characterize the material. This coefficient Z reflects the maximum ΔT obtainable with a material at zero cooling power, while heat pumps operate at a ΔT well below the maximum, the object of this simplified analysis is to show how heat pumps performances vary with the components of Z.

$$Z = \frac{a^2}{\rho . \lambda}$$

a Seebeck coefficient, ρ electrical resistivity, λ thermal conductivity.
To calculate the influence of the above coefficients, it is necessary to examine a heat pump of which the characteristics are given in Table 1.

Table 1

	Units	General	Auxilary circuit	Primary circuit
Surface of thermoelectric material	m2	, 0.23		
Thermal conductance	$W.K^{-1}$		0.46	0.99
Air flow rate	$kg.s^{-1}.cm^{-2}$ TE		$0.81.10^{-3}$	$3.23.10^{-3}$
Inlet temperature	° C ; %		0° C 70 % HR	12.8°C 36 % HR
Current density	$A.cm^{-2}$	80		

To examine the influence of these coefficients a, ρ and λ and the resulting coefficient of merit Z, a material which is available is used as a reference point.

$a = 203.10^{-6}$ $V.K^{-1}$

$\rho = 10.10^{-6} \Omega.m$

$\lambda = 1.44$ $W.m^{-1}$ K^{-1}

$$Z = \frac{a^2}{\rho . \lambda} = 2.86 \; 10^{-3} \; K^{-1}$$

The two main characteristics of a heat pump are for a given configuration : heating power and COP, so these two characteristics are plotted versus Z.
The curves are parametered in a. The three following values of a are chosen.

$a = 203.10^{-6}$ $V.K^{-1}$

$a = 244.10^{-6}$ $V.K^{-1}$

$a = 293.10^{-6}$ $V.K^{-1}$

It is necessary to make assumptions on the relationship between λ and ρ as a function of Z. The following two simple relationships have been assumed.
In Fig. 6 the solid lines are curves plotted at constant thermal conductivity λ = 1.44 $W.m^{-1}$ K^{-1} and the dashed lines are curves plotted at constant electrical resistivity $\rho = 10.10^{-6} \Omega.m$. These curves are plotted for a constant current density J ($A.cm^{-2}$). It is necessary to also know how P_H and COP vary as a function of J. The variations can be linearized for small changes for J of the order of ± 15 A. cm^{-2}. One obtains at Z = $2.86.10^{-3}$ K^{-1}. For values of J between 65 and 95 $A.cm^{-2}$

$$\frac{\Delta P_H}{\Delta J} = + 150 \; w.A^{-1} \; cm^2$$

$$\frac{\Delta COP}{\Delta J} = - 10^{-3} \; A^{-1}.cm^2$$

The variations are examined for Z = $2.86.10^{-3}$ K^{-1}.
Fig. 6 where the thermal conductivity is kept constant shows that when COP is not a major factor if the variation is kept below 0.05, an increase of a from 203.10^{-6} $V.K^{-1}$ to 244.10^{-6} $V.K^{-1}$ brings an increase of heating power of 35 %. Such an increase represents for a given heating power an important reduction in materials and in manufacturing cost.
Fig. 6 where the electrical resistivity is kept constant shows that variations in the Seebeck coefficient a, have a negligible influence on the heating power and an increase in a from 203.10^{-6} $V.K^{-1}$ to 244.10^{-6} $V.K^{-1}$ decreases the COP by 9 %.
Only one example where Z increases is examined : when Z goes from $2.86.10^{-3}$ K^{-1} to 3.5 x 10^{-3} K^{-1}.

Assuming a goes from 203 to 293.10^{-6} V.K^{-1}
with λ = constant

. P_H goes from 5.098 kW to 8.740 kW

. COP remains constant at 1.35

with ρ = constant

. P_H goes from 5.098 to 5.373

. COP drops from 1.357 to 1.261

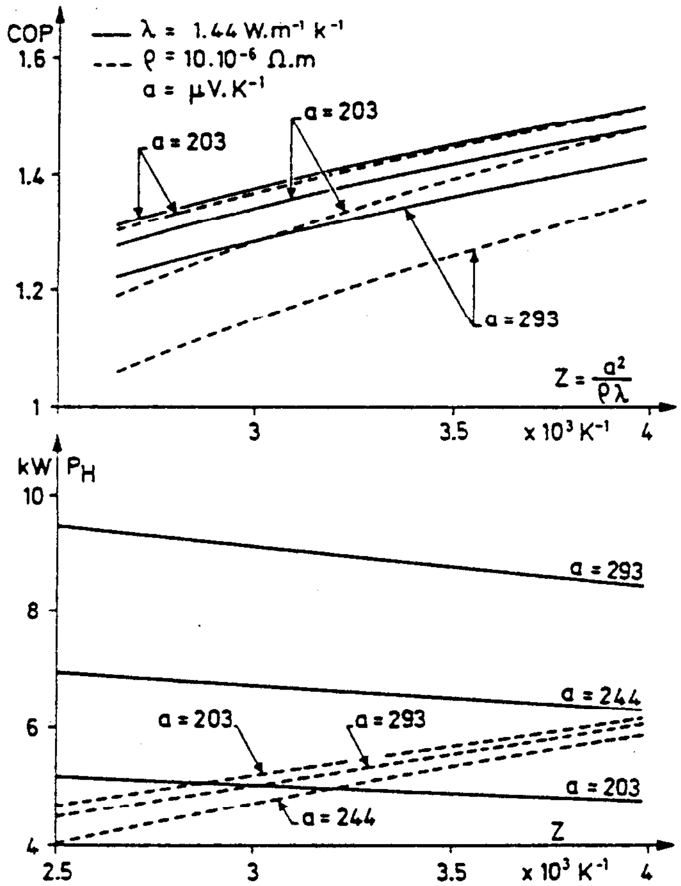

Fig. 6 - Influence of thermoelectric material characteristics on heating power P_H and COP.

4. <u>Frosting in auxilary circuit</u>

The problem of frosting in heat pumps using a compression cycle requires frequent defrosting procedures. In the case of thermoelectric heat pumps, the temperatures differences between heat exchanger surfaces and air are smaller, nevertheless frosting can occur. The following system is examined in Table 2.

Table 2

	Units	General	Primary circuit	Auxilary circuit
Surface of thermoelectric material	m2	0.23		
Thermal conductances	WK^{-1}		0.98	0.50
Air flow rates	kg.s^{-1}		0.24	0.41
Inlet temperature	° C, % RH		9,5 ; 72	- 4 ; 90
Current density	A.cm^{-2}	107		

The mathematical model with the inlet conditions given in Table 2 enables the calculation all over the system of the temperature of the heat exchanger surface in contact with air.

The conditions for frosting are twofold :
- fin temperature must be below 0° C.
- fin temperature must be below dew point temperature of air.

Fig. 7 gives the proportion of LA which is frosted as a function of LP.

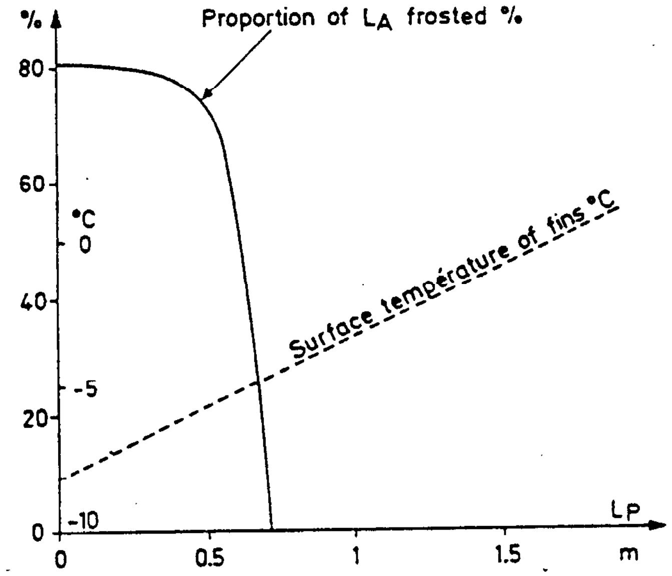

Fig. 7 - Proportion of L_A frosted and fin temperature versus L_P.

Figure 7 shows that for the conditions given in Table 2 only the fins up to L_P = 0.65 m will frost.

It is important to know the proportion of fins in the whole system that are frosted as a function of outside temperature. Figure 8 gives the proportion of fin area that is frosted as a function of outside temperature. The system is the one of Table 2 except that J is varied as a function of the outside temperature because in the case examined the heat requirements varied with the outside temperature.

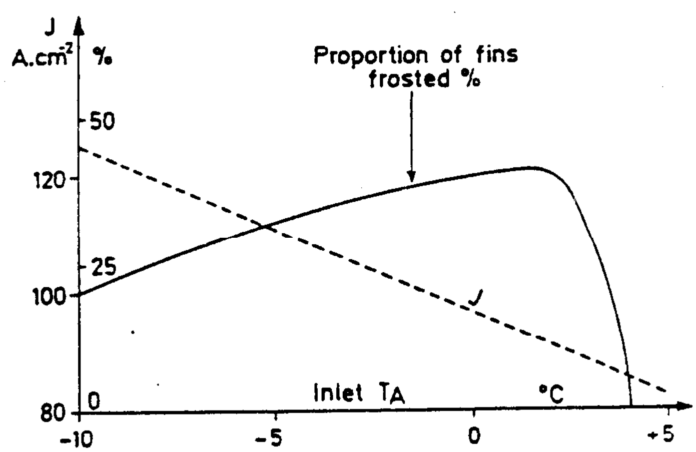

Fig. 8 - Proportion of fins frosted as a function of inlet temperature of auxilary circuit.

The above graph shows that maximum frosting occurs between - 1° C and + 3° C. Systems different to this one will give slightly different curves.

5 - <u>Industrial unit</u>

In the previous paragraphs, the parameters and their influence on performance have been examined especially in the range of auxilary air temperatures around 0° C, where the COP is often near to unity.

Thermoelectric heat pumps have contraints that vary with the industrial application. They are for example :

- Cost
- C.O.P.
- Volume and mass
- Materials that must resist to corrosive atmospheres
- etc...

An industrial unit is here described that is designed to operate in the heating and the cooling mode. Its main characteristics are given in Table 3.

Table 3

	Units	General	Primary circuit	Auxilary circuit
Overall dimensions	m	height L_p = 0.5	1.3	0.66
Surface of thermoelectric material	m2	0.20		
Thermal conductivity of heat exchangers	$W.K^{-1}$		0.98	0.46
Air flow rate	$kg.s^{-1}$		0.29	0.45
Inlet conditions	° C, % RH		12.8 ; 36	0 ; 70
Current density	$A.cm^{-2}$	50 to 130		

The temperature conditions chosen are those where the temperature differences between the two gas circuits are large, so as to see the extreme conditions where heat pumps have a COP only slightly greater than one.

A photograph of the unit is given in Figure 9.

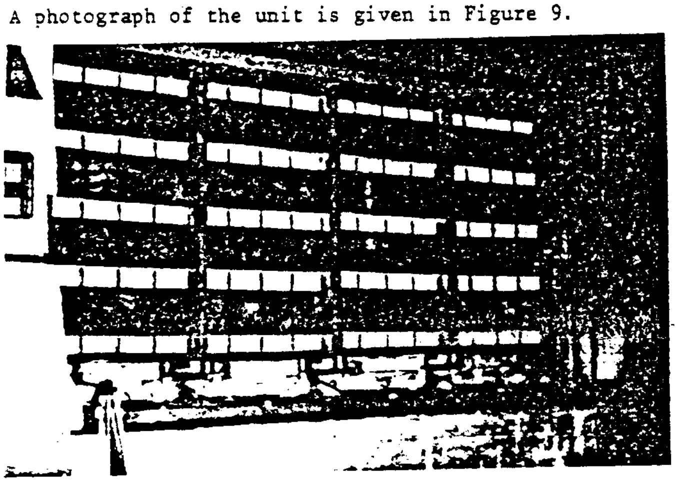

Fig. 9 - Photograph of unit.

Many diagrams can be used to present results. In thermoelectrics 3 dimensional diagrams are often very useful but require tremendous computations. The most two important graphs are for a given configuration and inlet conditions.

- Heating power per unit length of primary circuit versus length of primary circuit.
- Heating power and COP versus electrical current density J.

Figure 10 shows how the heating power $P_H.m^{-1}$ of L_P decreases as L_P increases. The decrease in this case is about 15 %. It is considered acceptable.

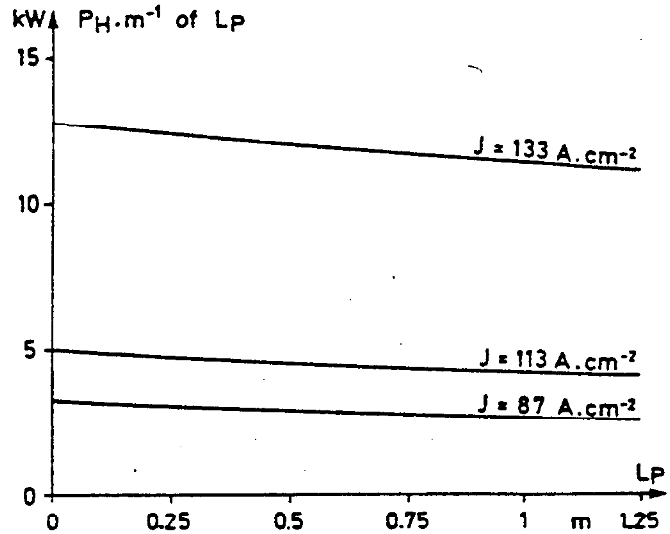

Fig. 10 - Heating power per unit length of L_P.

Having checked that the length L_P is acceptable, the procedures presented earlier can be applied to obtain L_A.

The most important graph is the one that gives heating power and COP versus J see Fig. 11.

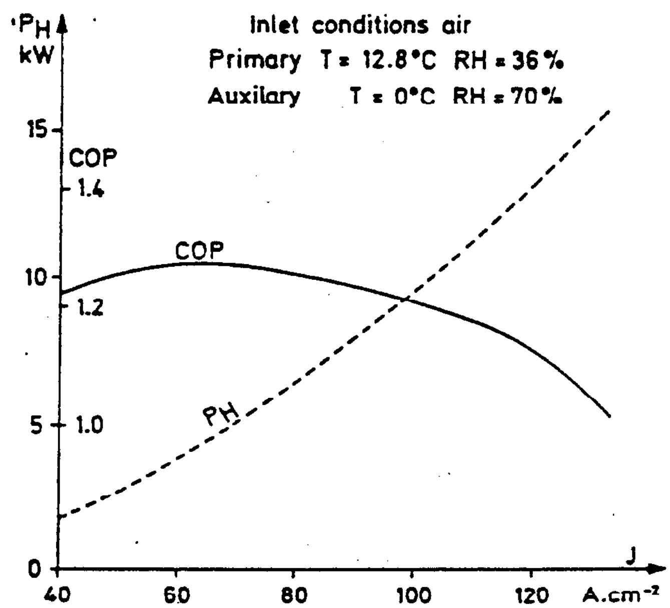

Fig. 11 - Heating power P_H and COP versus J.

This graph enables the choice of J depending on the heat requirements and the electric power supply losses, so that the overall COP is still greater than unity. It can be in certain cases beneficial in such a case to add some Joule heating of the auxilary air before introduction in the heat pump.

6 - Conclusions

A mathematical model has been presented and used to determine the limitations of thermoelectric heat pumps with low inlet temperatures of the auxilary circuit. The difference in temperature ($T_P - T_A$) considered is 12.8° C, with a COP of the order of 1.2. The COP can only be increased with improved thermoelectric materials. The heating power can be increased by using the higher thickness of the thermoelectric material without noticeably decreasing the COP.

Frosting in the configuration presented is less serious than for traditional heat pumps using a fluid compression cycle. Defrosting of thermoelectric heat pumps could be 10 times less frequent than for heat pumps with a compression cycle.

NOMENCLATURE

Symbol	Unit	Designation
a	$V.K^{-1}$	Seebeck coefficient
C	$W\,K^{-1}$	Thermal conductance
c	$J.kg^{-1}.K^{-1}$	Specific heat of gas
e	mm	Thickness of thermoelectric material (parallel to electrical current)
G	$kg.s^{-1}$	Mass flow rate of gas through unit
H	$W.m^{-2}\,K^{-1}$	Heat transfer coefficient $H = M.V^N$
I	A	Electrical current
J	$A.cm^{-2}$	Electrical current density
L	m	Length
M	$W.m^{-3}K^{-1}S^{-1}$	Constant in heat transfer coefficient
N	none	Exponent in heat transfer coefficient
P	W	Thermal power available in gas
P^*	none	Ratio of thermal power available in gas divided by aerodynamic power in primary circuit
Q	$m^3\,s^{-1}$	Volume flow rate of gas
r		Electrical resistance
$\bar{R}$	W	Joule losses
S	m2	Surface area
T	K	Degree Celsius
V	$m\,s^{-1}$	Gas velocity
V_E	m^{-3}	Volume of thermoelectric material
Z	K^{-1}	Coefficient of merit of thermoelectric material $= a^2 \rho^{-1} \lambda^{-1}$
ΔP	P	Pressure drop along a circuit
ΔT	K	Temperature difference
ρ	Ω . m	Electrical resistivity of thermoelectric material (T.E.)
λ	$W.K^{-1}$	Thermal conductivity of T.E. material
ϖ	$J.kg^{-1}$	Specific heat of gas at given temperature and pressure

Indices

P	Primary circuit) Preceded the other) indices
A	Auxilary circuit)
a	Relative to air
C	Cold side
H	Hot side
B	Heat exchanger base
D	Dimension perpendicular to P and A
E	T.E. material, interface or across material
F	Interface between heat exchanger and gas
L	Losses excluding T.E. material
1	Inlet conditions to unit
2	Exit conditions to unit

Convention

P_c	W	is negative
P_H	W	is positive
*		none dimensional
¯	W	Quantities with a bar above have the dimensions of a power

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